August 2, 2000

LBL-46431
UCB-PTH/0023
MPI-PHt/2000-26

The Lower Bound on the Neutralino-Nucleon Cross Section *

Vuk Manic[1], Aaron Pierce[1,2], Paolo Gondolo[3], and Hitoshi Murayama[1,2]

1. Theoretical Physics Group
Lawrence Berkeley National Laboratory
University of California
Berkeley, California 94720, USA
2. Department of Physics
University of California
Berkeley, CA 94720, USA
3. Max-Planck-Institut für Physik
Föhringer Ring 6, D-80805
München, Germany

Abstract

We examine if there is a lower bound on the detection cross section, $\sigma_{\chi-p}$, for the neutralino dark matter in the MSSM. If we impose the minimal supergravity boundary conditions as well as the "naturalness" condition, in particular $m_{1/2} < 350$ GeV, we show that there is a lower bound of $\sigma_{\chi-p} > 10^{-46}$ cm^2. We also clarify the origin for the lower bound. Relaxing either of the assumptions, however, can lead to much smaller cross sections.

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*The work of HM and AP was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grant PHY-95-14797. AP is also supported by a National Sicence Foundation Graduate Fellowship. The work of VM was supported by the Center for Particle Astrophysics, a NSF Science and Technology Center operated by the University of California, Berkeley, under Cooperative Agreement No. AST-91-20005 and by the National Science Foundation under Grant No. AST-9978911.

Disclaimer

1 Introduction

Supersymmetry is considered to be a compelling extension to the Standard Model for several reasons. For example, it stabilizes scalar masses against radiative corrections, allowing theories with fundamental scalars to become natural. For a review see [1]. Supersymmetry also weighs in on the dark matter problem: stars and other luminous matter contribute a small fraction of the critical density, $\Omega_{lum} = (0.003 \pm 0.001)h^{-1}$, while the amount of matter known to exist from its gravitational effects (both at galaxy and cluster of galaxies scales) is much larger, $\Omega_M = 0.35 \pm 0.07$ (Ref. [2]). Furthermore, most of the missing matter seems to be non-baryonic in nature. The experimental motivation behind the dark matter problem and different search strategies are discussed in more detail in [3, 4, 5].

In supersymmetric models, R-parity is often imposed to avoid weak-scale proton decay or lepton number violation. Imposing this symmetry also yields an ideal fermionic dark matter candidate. Namely, in supersymmetric models with R-parity, the lightest supersymmetric particle (LSP) is stable and it could conceivably make up a substantial part of the dark matter in the galactic halo.

Here we investigate the direct detection of such a particle. There have been many such studies in the literature [4, 6, 7, 8, 9, 10]. More recently, there have been discussions of numerous variables that can effect direct detection. These studies include an investigation of the effect of the rotation of the galactic halo [11], the effects of the uncertainty of the quark densities within the nuclei [12, 13], possible CP violation [14, 15], and non-universality of gaugino masses [12, 16]. Here we attempt to address a simpler question. Is there a minimum cross section for the elastic scattering of neutralinos off of ordinary matter? Naively, it would seem that a judicious choice of parameters might allow a complete cancellation between different diagrams. After all, the parameter space is very large in the general Minimal Supersymmetric Standard Model (MSSM), and even for a very restrictive framework such as the minimal supergravity (mSUGRA), the number of parameters is still quite large. We will show that there nonetheless exists a minimum cross section in the mSUGRA framework. However, we will also show that this result strongly depends on the assumptions of the framework, such as unification of different parameters at the GUT scale, radiative electroweak symmetry breaking and naturalness.

This argument is of great importance when considering the upcoming direct detection experiments. For the mSUGRA framework, one expects that the future ambitious direct detection experiments can explore most of the parameter space. However, we find that the detection picture is not quite as rosy for a more general MSSM framework.

2 Definitions and Approach

We adopt the following notation for the superpotential and soft supersymmetry breaking potential in the MSSM:

$$\begin{aligned} W &= \epsilon_{ij}(-\hat{e}_R^* h_E \hat{l}_L^i \hat{H}_1^j - \hat{d}_R^* h_D \hat{q}_L^i \hat{H}_1^j \\ &\quad + \hat{u}_R^* h_U \hat{q}_L^i \hat{H}_2^j - \mu \hat{H}_1^i \hat{H}_2^j), \end{aligned} \tag{1}$$

$$\begin{aligned} V_{soft} &= \epsilon_{ij}(\widetilde{e}_R^* A_E h_E \widetilde{l}_L^i H_1^j + \widetilde{d}_R^* A_D h_D \widetilde{q}_L^i H_1^j \\ &\quad - \widetilde{u}_R^* A_U h_U \widetilde{q}_L^i H_2^j - B\mu \hat{H}_1^i \hat{H}_2^j + h.c.) \\ &\quad + H_1^{i*} m_{H_1}^2 H_1^i + H_2^{i*} m_{H_2}^2 H_2^i \\ &\quad + \widetilde{q}_L^{i*} M_Q^2 \widetilde{q}_L^i + \widetilde{l}_L^{i*} M_L^2 \widetilde{l}_L^i + \widetilde{u}_R^* M_U^2 \widetilde{u}_R \\ &\quad + \widetilde{d}_R^* M_D^2 \widetilde{d}_R + \widetilde{e}_R^* M_E^2 \widetilde{e}_R \\ &\quad + \frac{1}{2}(M_1 \widetilde{B}\widetilde{B} + M_2 \widetilde{W}^a \widetilde{W}^a + M_3 \widetilde{g}^b \widetilde{g}^b). \end{aligned} \tag{2}$$

Here the h's are Yukawa couplings, the A's are trilinear couplings, the $M_{Q,U,D,L,E}$ are the squark and slepton mass parameters, the $M_{1,2,3}$ are gaugino mass parameters and m_{H_1}, m_{H_2}, μ, and B are Higgs mass parameters. The i and j are $SU(2)_L$ indices, and are made explicit, so as to make our sign conventions clear. $SU(3)$ indices are suppressed. In the R-parity invariant MSSM the LSP is usually a neutralino - a mixture of bino, neutral wino and two neutral higgsinos. In our notation, the neutralino mass matrix reads

$$\begin{pmatrix} M_1 & 0 & -m_Z s_{\theta_W} c_\beta & +m_Z s_{\theta_W} s_\beta \\ 0 & M_2 & +m_Z c_{\theta_W} c_\beta & -m_Z c_{\theta_W} s_\beta \\ -m_Z s_{\theta_W} c_\beta & +m_Z c_{\theta_W} c_\beta & 0 & -\mu \\ -m_Z s_{\theta_W} s_\beta & -m_Z c_{\theta_W} s_\beta & -\mu & 0 \end{pmatrix} \tag{3}$$

Here $s_\beta = \sin\beta$, $c_\beta = \cos\beta$, $s_{\theta_W} = \sin\theta_W$, and $c_{\theta_W} = \cos\theta_W$. The physical states are obtained by diagonalizing this matrix. The lightest neutralino can be written in the form:

$$\chi_1^0 = N_{11}\tilde{B} + N_{12}\tilde{W}_3 + N_{13}\tilde{H}_1^0 + N_{14}\tilde{H}_2^0. \tag{4}$$

We are interested in spin independent scattering of neutralinos off of ordinary matter. This contribution dominates in the case of detectors with large nuclei, such as Ge [17]. As discussed in the literature, in most situations the dominant contribution to the spin independent amplitude is the exchange of the two neutral Higgs bosons, although in some cases the contribution of the squark exchange and loop corrections are substantial. The relevant tree-level diagrams are shown in Fig. 1.

We use the DarkSUSY package to evaluate the cross section [18]. The code has the following inputs: $M_{1,2,3}$, μ, the ratio of the vevs of the two Higgs bosons ($\tan\beta = v_2/v_1$), the mass of the axial Higgs boson (m_A), the soft masses of the sparticles ($M_{Q,U,D,L,E}$) and the diagonal components of the trilinear coupling matrices ($A_{E,D,U}$). All inputs are to be supplied at the weak scale. DarkSUSY then calculates the particle spectrum, widths and couplings based on the input parameters. It evaluates the cross section for scattering of neutralinos off protons and neutrons, following Ref. [10]. It also evaluates the relic density of the neutralinos for the given input parameters following [19], which includes the relativistic Boltzmann averaging, subthreshold and resonant annihilation and coannihilation processes with charginos and neutralinos. Furthermore, DarkSUSY checks for the current constraints obtained by experiments, including the $b \to s + \gamma$ constraint [20, 21].

3 mSUGRA Framework

3.1 Definition of the Framework

In this section, we briefly outline the mSUGRA framework, and then discuss the results of the calculation. In mSUGRA, one makes several assumptions:

- There exists a Grand Unified Theory (GUT) at some high energy scale. Consequently, the gauge couplings unify at the GUT scale. The value of the couplings at the weak scale determines the GUT scale to be $\approx 2\times 10^{16}$ GeV. The gaugino mass parameters also unify to $m_{1/2}$ at the GUT scale.

- Other unification assumptions are: the scalar mass parameters unify to a value denoted by m_0 and the trilinear couplings unify to A_0 at the GUT scale. Using the MSSM renormalization group equations (RGE's) we evaluate all parameters at the weak scale. We choose to do this using the one loop RGE's that can be found, for example, in [22] or [23].

- Radiative electroweak symmetry breaking (REWSB) is imposed: minimization of the one-loop Higgs effective potential at the appropriate scale fixes μ^2 and m_A (we follow the methods of Refs. [24] and [25]). For completeness, we reproduce the equation for μ^2 at tree level:

$$\mu^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2\beta}{\tan^2\beta - 1} - \frac{1}{2}M_Z^2. \tag{5}$$

With these assumptions, the mSUGRA framework allows four free parameters (m_0, $m_{1/2}$, A_0 and $\tan\beta$). Also, the sign of μ remains undetermined. Starting with these parameters we determine all of the input parameters for the DarkSUSY code. We allow the free parameters to vary in the intervals

$$\begin{aligned} &0 < m_{1/2} < 350\,\mathrm{GeV},\ 95 < m_0 < 1000\,\mathrm{GeV}, \\ &-3000 < A_0 < 3000\,\mathrm{GeV},\ 1.8 < \tan\beta < 25. \end{aligned} \tag{6}$$

We choose the constraint on $m_{1/2}$ such that the constraint on the gluino mass is $M_3 < 1$ TeV. Later, we will examine the effects of relaxing this constraint to $0 < m_{1/2} < 1000$ GeV, corresponding to $0 < M_3 \leq 3000$ GeV. The upper limit on these parameters comes from the naturalness assumption: one of the reasons for using supersymmetry is its ability to naturally relate high and low energy scales; as a result, no parameter in the theory should be very large. Note that most naturalness constraints quoted in the literature are more stringent than ours. The low value of $\tan\beta$ is set by the requirement that the top Yukawa coupling does not blow up before the GUT scale is reached.

Before we present the detailed analysis of cross section, a few remarks are in order. First, the $b \to s + \gamma$ constraint eliminates large portions of the $\mu < 0$

parameter space, in agreement with [26], [27]. Second, for both $\mu > 0$ and $\mu < 0$, we find no higgsino-like LSP models that are cosmologically important, in agreement with [27].

We plot the variation of the spin independent cross section versus the neutralino mass in Fig. 2. The upper bound on σ (of the theoretically allowed regions) comes from the lower bound on the relic density (we use the rather conservative $0.025 < \Omega h^2$). The lower bound on M_χ comes from the existing constraints from the accelerator experiments. The upper bound on M_χ is a combination of the upper bound on relic density (again conservative $\Omega h^2 < 1$) and of the bounds on the free parameters. The lower bound on σ is not yet well understood, and it is the subject of this paper. Notice that a similar plot has already appeared - Fig. 1 in Ref. [12]. We find good agreement with this reference. Figure 2 also includes some recent and future direct detection experimental results [28, 29, 30, 31, 32]. Note that the parameter space defined by Eq. (6) corresponds to the region of $\sigma_{\chi-p}$ - M_χ plane bounded by the closed dashed line. Therefore, $\sigma > 10^{-46}\text{cm}^2$ for these models, or equivalently, assuming a ^{73}Ge target, the dark matter density $\rho_D = 0.3$ GeVc^{-2}cm^{-3}, the WIMP characteristic velocity $v_0 = 230$ km s^{-1} and following Ref. [33], the event rate $R > 0.1$ ton^{-1}day^{-1}. Hence, the most ambitious future direct detection experiments may be able to explore a large portion, if not all, of the these models.

3.2 Results and Analysis

As mentioned above, the dominant contribution to spin independent elastic scattering is usually the Higgs boson exchange. Figure 3 illustrates this relationship within our results. For this figure we used the less restrictive naturalness assumption $0 < m_{1/2} < 1$TeV, which we will consider in more detail later; this constraint includes all points defined by Eq. (6). The nearly perfect 45 degree line in the figure indicates good agreement between the total cross section as evaluated by DarkSUSY and the cross section calculated including the exchange of Higgs bosons only (in the approximation explained below). We will, therefore, concentrate on the Higgs boson exchange and will postpone the discussion of squark exchange to the end of this section.

The contribution of Higgs boson exchange can be found in the literature [4, 10, 12, 14, 34, 35, 36]. It is of the following form:

$$\sigma_{h,H} \sim |f_u A^u + (f_d + f_s) A^d|^2, \tag{7}$$

where $f_u \approx 0.021, f_d \approx 0.029, f_s \approx 0.21$ parametrize the quark-nucleon matrix elements and

$$\begin{aligned} A^u &= \frac{g_2^2}{4M_W}\Big(\frac{F_h}{m_h^2}\frac{\cos\alpha_H}{\sin\beta} + \frac{F_H}{m_H^2}\frac{\sin\alpha_H}{\sin\beta}\Big), &(8)\\ A^d &= \frac{g_2^2}{4M_W}\Big(-\frac{F_h}{m_h^2}\frac{\sin\alpha_H}{\cos\beta} + \frac{F_H}{m_H^2}\frac{\cos\alpha_H}{\cos\beta}\Big), &(9)\end{aligned}$$

and

$$\begin{aligned} F_h &= (N_{12} - N_{11}\tan\theta_W)(N_{14}\cos\alpha_H + N_{13}\sin\alpha_H)\\ F_H &= (N_{12} - N_{11}\tan\theta_W)(N_{14}\sin\alpha_H - N_{13}\cos\alpha_H). &(10)\end{aligned}$$

The N's are the coefficients appearing in Eq. (4) and α_H is the Higgs boson mixing angle (defined after radiative corrections have been included in the Higgs mass matrix). A^u represents the amplitude for scattering off an up-type quark in a nucleon, while A^d represents the amplitude for scattering off a down-type quark in the nucleon. Note that there is an upper bound on the light Higgs boson mass in the MSSM, given by $m_h < 130$ GeV [37]. Since all models we generate have a bino-like neutralino, $\mu > M_1$ and $\mu > M_Z$. Then, following Ref. [34], we can expand the N_{1i}'s out in powers of $\frac{M_Z}{\mu}$. We reproduce their result here:

$$\begin{aligned} N_{11} &\approx 1, &(11)\\ N_{12} &\approx -\frac{1}{2}\frac{M_Z}{\mu}\frac{\sin 2\theta_W}{(1 - M_1^2/\mu^2)}\frac{M_Z}{M_2 - M_1}\left[\sin 2\beta + \frac{M_1}{\mu}\right],\\ N_{13} &\approx \frac{M_Z}{\mu}\frac{1}{1 - M_1^2/\mu^2}\sin\theta_W\sin\beta\left[1 + \frac{M_1}{\mu}\cot\beta\right],\\ N_{14} &\approx -\frac{M_Z}{\mu}\frac{1}{1 - M_1^2/\mu^2}\sin\theta_W\cos\beta\left[1 + \frac{M_1}{\mu}\tan\beta\right].\end{aligned}$$

First of all, let us check if the couplings F_h and F_H can be made arbitrarily small (i.e. if $F_h = 0$ and/or $F_H = 0$ are possible). We distinguish four cases.

Fig. 4 illustrates which regions of the M_1-μ plane satisfy the conditions of the four cases. In the following, we will use the approximation of Eq. (11).

- Case 1: $N_{12} - N_{11}\tan\theta_W = 0$ would make both F_h and F_H vanish. Intuitively, this is reasonable since this condition implies that the neutralino is a pure photino, and the tree level Higgs coupling to the photino vanishes. Using Eq. (11), we may rewrite this condition as

$$\mu(M_2 - M_1) = -M_Z^2\cos^2\theta_W\left(\sin 2\beta + \frac{M_1}{\mu}\right). \tag{12}$$

Since $M_2 > M_1 > 0$, the last equation can be satisfied only if $\mu < 0$. If we use the GUT relationship betweeen M_1 and M_2, we get

$$\mu M_1\left(\frac{3\cos^2\theta_W}{5\sin^2\theta_W} - 1\right) = -M_Z^2\cos^2\theta_W\left(\sin 2\beta + \frac{M_1}{\mu}\right) \tag{13}$$

or, equivalently, for μ and M_1 in units of GeV

$$M_1 = \frac{-6449\sin 2\beta}{\mu + \frac{6449}{\mu}}. \tag{14}$$

As $\sin 2\beta$ ranges from 0 to 1, Eq. (14) spans the dashed regions marked 'Case 1' in Fig. 4. For $\sin 2\beta = 1$, Eq. (14) implies $M_\chi \approx M_1 < 40$ GeV. This can be read directly from Fig. 4. Note that such low values of M_χ are excluded by the relic density constraint and current experimental limits (as shown in Fig. 2 and Ref. [12]). The constraint on M_χ is even stronger for other values of $\sin 2\beta$, so we conclude that this condition cannot be satisfied in the mSUGRA framework.

- case 2: $N_{14}\sin\alpha_H - N_{13}\cos\alpha_H = 0$ would make only F_H vanish. In our approximation, this condition translates into

$$M_1/\mu = -\cot\alpha_H - \cot\beta. \tag{15}$$

To understand the meaning of this condition better, we will use the tree level relationship between α_H and β:

$$\cot 2\alpha_H = k\cot 2\beta,\ k = \frac{m_A^2 - M_Z^2}{m_A^2 + M_Z^2}, \tag{16}$$

which, after some trigonometric manipulations, yields

$$\begin{aligned}\cot\alpha_H &= \frac{k}{2}\left(\frac{1}{\tan\beta} - \tan\beta\right) \\ &\quad -\sqrt{\frac{k^2}{4}\Big(\frac{1}{\tan^2\beta} + \tan^2\beta - 2\Big) + 1}. \end{aligned} \tag{17}$$

Since both terms on the right hand side of Eq. (17) are negative because $\tan\beta > 1$, the minimum of $|\cot\alpha_H| = 1$ occurs when $k = 0$ (or, equivalently, when $m_A = M_Z$). Then, since $\tan\beta > 1.8$, the condition of Eq. (15) becomes

$$\frac{M_1}{\mu} > 0.5. \tag{18}$$

At this point, we would like to formulate a relationship between μ and M_1 resulting from the RGEs and REWSB assumptions. In Ref. [23], an approximate solution (based on the expansion around the infrared fixed point) to the RGEs for the Higgs mass parameters, m_{H_1} and m_{H_2}, is presented. Assuming that the value of the top Yukawa coupling is relatively close to the infra-red fixed point, we can write:

$$\begin{aligned} m_{H_1}^2 &\approx m_0^2 + 0.5m_{1/2}^2, \\ m_{H_2}^2 &\approx -0.5m_0^2 - 3.5m_{1/2}^2. \end{aligned} \tag{19}$$

These equations, coupled with Eq. (5), yield a value for μ^2 in terms of $m_{1/2}, m_0$, and $\tan\beta$. Using the GUT relationship:

$$M_1 = m_{1/2}\frac{\alpha_1(m_Z)}{\alpha_{GUT}}, \tag{20}$$

we get a roughly linear relationship between μ and M_1:

$$M_1 = (0.3|\mu| - 60) \pm 40. \tag{21}$$

The spread ± 40 comes from the variation in m_0 and $\tan\beta$ and the linearity breaks down somewhat at the low values of M_1. The empirical relationship that we obtain from running the code (see Fig. 4) is very similar:

$$M_1 = (0.3|\mu| - 40) \pm 25. \tag{22}$$

The smaller spread comes from the application of the relic density cut and the current experimental limits. In any case, we conclude that $M_1/\mu > 0.3$ is not allowed in the mSUGRA framework. This is in conflict with Eq. (18), implying that the Case 2 cannot be satisfied. Note that the tree level relationship in Eq. (16) is altered at higher orders, but we have checked that this does not affect the final conclusion.

- Case 3: $N_{14}\cos\alpha_H + N_{13}\sin\alpha_H = 0$ would make F_h vanish. Manipulation of this condition using Eq. (11) yields

$$M_1/\mu = \tan\alpha_H - \cot\beta. \tag{23}$$

  Figure 5 shows that when this condition is (approximately) satisfied, the elastic scattering cross section is dominated by the heavy Higgs boson exchange and the light Higgs boson exchange contribution is indeed much smaller. Hence, even if this condition is satisfied, σ cannot be arbitrarily small due to heavy Higgs boson exchange. Of course, this assumes that there is some upper bound on the heavy Higgs mass, which is true simply because the parameter space is bounded. On the other hand, since $\tan\alpha_H < 0$, the condition for the vanishing of the light Higgs boson contribution can be satisfied only for $\mu < 0$. However, the $b \to s + \gamma$ constraint (along with $0 < m_{1/2} < 350$ GeV), excludes most of the $\mu < 0$ models (in particular all of the models shown on Fig. 5). Therefore, this constraint eliminates all models which could satisfy the condition in Eq. (23). In this way, the $b \to s + \gamma$ constraint keeps the light Higgs boson exchange important, which keeps the cross section relatively accessible to direct detection experiments.

- Case 4: There is one more way of making both F_h and F_H small, and that is by making μ very large (N_{12}, N_{13}, and N_{14} all contain μ in denominator). However, this possibility is limited by naturalness assumption: μ is kept below ≈ 900 GeV by the upper bound we have chosen on $m_{1/2}$. Hence, in mSUGRA framework, the naturalness assumption also keeps the cross section from vanishing.

We conclude, therefore, that with the definition of Eq. (6) F_h and F_H cannot be arbitrarily small.

We now consider whether different contributions add constructively or destructively. Let us examine the relative signs of F_h, F_H and μ. Since $\tan\beta > 1.8$ and $M_1/\mu \approx 0.3$, it follows from Eq. (11) that $|N_{13}| > |N_{14}|$. Then, since $\cot\alpha_H < -1$ (and $\sin\alpha_H < 0$), the N_{13} term dominates over the N_{14} term in F_H (Eq. (10)). Hence, $F_H/\mu > 0$ always, consistently with the analysis of Case 2 above. The situation is somewhat more complicated in case of F_h. For $\mu > 0$, following similar analysis we get $F_h/\mu > 0$. Then, the interference between the two terms in A^u (Eq. (8)) is destructive and the interference between the two terms in A^d (Eq. (9)) is constructive. Furthermore, in Eq. (7) we see that A^u gets multiplied by a much smaller form factor than A^d. As a result, A^d strongly dominates over A^u in Eq. (7), preventing σ from vanishing.

On the other hand, for $\mu < 0$ and for sufficiently large $\tan\beta$, N_{14} can change sign. As we will see in the case of a more general MSSM framework (next section), this could lead to cancellations of different parts of the amplitude and to very low values of σ. However, as mentioned above, in mSUGRA framework most of the $\mu < 0$ points are excluded by the $b \to s + \gamma$ constraint, along with the upper bound on $m_{1/2} < 350$ GeV. We conclude that in the parameter space defined by Eq. (6), A^d always dominates over A^u, so σ does not vanish.

The situation changes significantly if we expand the parameter space slightly. Instead of $m_{1/2} < 350$ GeV, we now impose $m_{1/2} < 1$ TeV. The result is shown in Fig. 2. The effect of this change is two-fold. First, there are now models with $\mu < 0$ that pass the $b \to s + \gamma$ constraint. (Along the lines of [34], larger $m_{1/2}$ makes μ large and hence suppresses the charged Higgs contribution. Large $m_{1/2}$ also makes the stop mass larger, which in turn makes the $\widetilde{W} - \tilde{t}$ channel of the $b \to s + \gamma$ decay smaller.) Consequently, the condition of Case 3 above can be satisfied, while avoiding the experimental constraints. Then the contribution of the light Higgs boson exchange can be neglected and the heavy Higgs boson exchange dominates. Second, by Eqs. (20) and (22), μ can now have larger values, which makes F_H smaller, bringing σ further down. Therefore, the result is that relaxing the naturalness constraint by a factor of three pushes the lower bound on σ down by two orders of magnitude.

With the above discussion in hand, let us go back and consider the squark exchange. The complete calculation of the squark exchange contribution is fairly complex. However, good insights can be gained by making several simplifying assumptions. First of all, the contribution of the squark exchange can be roughly approximated by the contribution of the exchange of the u, d, and s squarks. In this case, the contribution of the squark exchange to the cross-section can be written as

$$\sigma_{\tilde{q}} \sim |f_u B_u + f_d B_d + f_s B_s|^2, \tag{24}$$

where f_i is as defined above, and the B_j represent the amplitude for scattering off of a quark of type j in the nucleon. Furthermore, in the following considerations we neglect the left-right mixing in these light squarks. This should be true over a large class of models, as the off-diagonal elements in the squark mass matrix are proportional to the corresponding quark mass. Let us also neglect the mass splitting of the two different squarks. Also, since $f_s \gg f_u$, f_d, we can neglect all but the B_s term. In this approximation, following Ref. [14], we can write:

$$B_s = -\frac{1}{4m_s}\frac{1}{M_{\tilde{s}}^2 - M_{\chi_1^0}^2}[2C_1C_2 - 2C_1C_3], \tag{25}$$

where we have defined the following:

$$\begin{aligned} C_1 &= \frac{g_2 m_s N_{13}}{2M_W\cos\beta}, \\ C_2 &= eQy_1 + \frac{g_2}{\cos\theta_W}y_2[T_3 - Q\sin^2\theta_W], \\ C_3 &= eQy_1 - \frac{g_2}{\cos\theta_W}y_2 Q\sin^2\theta_W. \end{aligned} \tag{26}$$

Note that C_1 represents the coupling of the down type quark to the Higgsino portion of the neutralino. C_2 and C_3 represent the couplings of bino to the left and right handed quark, respectively. Here, T_3 is the $SU(2)$ quantum number of the squark in question, Q is the charge, y_1 denotes the photino fraction of the neutralino, while y_2 denotes the zino fraction. They are given by:

$$\begin{aligned} y_1 &= N_{11}\cos\theta_W + N_{12}\sin\theta_W, \\ y_2 &= -N_{11}\sin\theta_W + N_{12}\cos\theta_W. \end{aligned} \tag{27}$$

After approximating $y_2 \approx -\sin\theta_W$ and using $\tan\theta_W = g'/g$, a brief and straight-forward calculation yields a simple expression for the amplitude due to the exchange of the strange squarks:

$$B_s = \frac{-g_2 g'}{8M_W\cos\beta}\frac{N_{13}}{M_{\tilde{s}}^2 - M_{\chi}^2}. \tag{28}$$

Furthermore, we can write the masses $M_{\tilde{s}}$ and M_χ in terms of the input parameters of mSUGRA. This is because the Yukawa couplings can be neglected in the RGEs. Following the methods described in Ref. [38], and using the Eq. (20) we can write

$$M_{\tilde{s}_R}^2 - M_\chi^2 \approx m_0^2 + 5.8 m_{1/2}^2. \tag{29}$$

Using Eqs. (9) and (28), we can compare the squark exchange to the light Higgs boson exchange:

$$\frac{A^d}{B_s} = \frac{2\sin\alpha_H (N_{14}\cos\alpha_H + N_{13}\sin\alpha_H)(m_0^2 + 5.8 m_{1/2}^2)}{m_h^2 N_{13}}. \tag{30}$$

Here we have only kept the contribution of the strange quark to the Higgs exchange amplitude (A^d) as well. Note that in general, the light Higgs boson contribution will dominate. As expected, this is basically due to the fact that squarks are in general heavier than the lightest Higgs boson. The squark exchange can be important only if the contribution from the exchange of the Higgs bosons is fine-tuned to be very small.

4 General MSSM Framework

4.1 Definition of the Framework

In this framework we relax our assumptions. We keep the unification of the gaugino masses, but we drop the requirements that the scalar masses and the trilinear scalar couplings unify. In addition, we drop the REWSB requirement (i.e., we take $m_{H_{1,2}}^2$ as independent parameters from m_0). We assume that all scalar mass parameters at the *weak scale* are equal: m_{sq}. This assumption is made in order to simplify the calculation, and it should not affect the general flavor of our results. Of all trilinear couplings, we keep only A_t and A_b and we set all others to zero. Then, the free parameters are $\mu, M_2, \tan\beta, m_A, m_{sq}, A_t, A_b$. We also relax the naturalness assumption, allowing the free parameters to have very large values. Besides the relatively uniform scans of the parameter space, we also performed special scans in order to investigate the different conditions mentioned in the previous section. The free parameter space is then:

$$-300\,\mathrm{TeV} < \mu < 300\,\mathrm{TeV}, \qquad 0 < M_2 < 300\,\mathrm{TeV},$$

$$95\,\mathrm{GeV} < m_A < 10\,\mathrm{TeV}, \qquad 200\,\mathrm{GeV} < m_{sq} < 50\,\mathrm{TeV},$$
$$-3 < \frac{A_{t,b}}{m_{sq}} < 3, \qquad 1.8 < \tan\beta < 100. \tag{31}$$

Again, a few comments are in order. First, in this framework we observe higgsino-like (as well as bino-like) lightest neutralino. In agreement with the Ref. [39], we find very few light higgsino-like models, which will probably be explored soon by accelerator experiments. Most of the higgsino-like models (with gaugino content $z_g < 0.01$) have $M_\chi > 450$ GeV, implying very large values of $m_{1/2}$. In particular, in higgsino like models $M_1 > \mu \approx M_\chi$; our results give $M_1 > 700$ GeV or, equivalently, $m_{1/2} > 1700$ GeV. These values are considered unnatural. For these reasons, we choose not to analyze the higgsino case. Second, $b \to s + \gamma$ is less constraining (allowing $\mu < 0$ bino-like models), but our results are still consistent with Refs. [26], [27].

We present the plot of σ versus M_χ in this framework (Fig. 6). We do not pretend that Fig. 6 reflects all points accessible in a general MSSM. However, it does serve to show some generic differences from the mSUGRA case. Namely, we can obtain much larger values for the neutralino mass because of the size of the parameter space. In addition, the lower bound on σ is also much lower than in the mSUGRA case. We discuss the specifics of this below.

4.2 Results and Analysis

We concentrate only on a bino-like lightest neutralino. All results in this section are presented with this assumption in mind. In particular, we demand $z_g = (N_{11}^2 + N_{12}^2)/(N_{13}^2 + N_{14}^2) > 10$. In this case, we can rely on the same approximations we used in the previous section. In particular, the expansion of Eq. (11) is valid. So, we can revisit the 4 different cases explored in the previous section.

- Case 1: $N_{12} - N_{11}\tan\theta_W = 0$ cannot be satisfied because, as in the mSUGRA case, it implies $M_\chi < 40$ GeV, which is ruled out by the relic density cut and the experimental limits (as shown on Fig. 6). The points that get close to satisfying this condition have very low contributions due to the Higgs bosons exchange, so this is one of the rare situations where the squark exchange is important.

- Case 2: $N_{14} \sin \alpha_H - N_{13} \cos \alpha_H = 0$ is now possible to satisfy because μ and M_1 are not related. We indeed observe that the heavy Higgs boson exchange contribution is very small in this case, but since the light Higgs boson exchange dominates, σ is kept relatively high in value.

- Case 3: $N_{14} \cos \alpha_H + N_{13} \sin \alpha_H = 0$ is indeed possible to satisfy. As in the mSUGRA case, the light Higgs boson exchange is small and heavy Higgs boson exchange dominates. Unlike in the mSUGRA case, $b \to s + \gamma$ does not disallow these models.

- Case 4: Since the naturalness constraint has been relaxed, μ is allowed to have very large values. Then N_{12}, N_{13}, and N_{14} can be driven small, which in turn would make the Higgs boson exchange contribution small. Intuitively, large $|\mu|$ implies that the neutralino is a very pure bino, for which the Higgs boson scattering channels vanish. If the squark masses are kept large as well, the squark contribution will be small too, making the total elastic scattering cross section very small. This is illustrated in Fig. 7 - the lowest values of σ are obtained for the largest values of $|\mu|$.

However, more importantly, there is a destructive interference that did not appear in mSUGRA framework. The $b \to s + \gamma$ is not as constraining in the MSSM framework, so models with $\mu < 0$ are allowed. Then, by Eq. (10), if $(\frac{M_1}{\mu} \tan \beta)$ is negative and sufficiently large, N_{14} can change sign. Following through Eqs. (10) and (9), this effect can induce destructive interference between terms in Eq. (9), so that A^d can be of opposite sign from A^u and relatively small in magnitude. Therefore, it is possible to have a destructive interference between A^d, A^u and the squark exchange amplitude resulting in very low values for σ.

In Table 1 we present some of the models in which this kind of cancellation takes place. Note that the values of the parameters in the Table (particularly in the last three rows) are not very large, illustrating that the naturalness alone cannot prevent this cancellation from happening. Similarly, there are models which have $|M_1/\mu| \approx 0.3$ and still obey this cancellation (for example, row 4 in the Table). Hence, REWSB assumption is also not sufficient in order to

μ (GeV)	M_1 (GeV)	m_{sq} (GeV)	m_A (GeV)	$\tan\beta$	A_t/m_{sq}	A_b/m_{sq}	σ (cm^2)
-1794	502	3792	1004	10.1	1.2	2.5	7.9×10^{-51}
-2109	534	3211	1087	11.8	-1.3	1.0	8.4×10^{-51}
-195	55	2995	1120	10.2	-2.0	2.3	2.1×10^{-50}
-182	61	2891	1099	7.0	-0.6	-0.1	1.6×10^{-50}
-274	163	325	1944	3.8	0.6	2.5	7.8×10^{-50}

Table 1: Some of the models in the general MSSM framework with very low values of $\sigma_{\chi-p}$.

avoid the cancellation. The effect is further enhanced (in the general MSSM framework) by the fact that $|M_1/\mu|$ is not fixed, as it was in mSUGRA case (by the REWSB and unification assumptions).

It is fair to ask, therefore, what prevents the cancellation from happening in the mSUGRA framework. We distinguish two cases.

First, we consider $M_\chi < 150$ GeV. As shown in the Fig. 2, all models in mSUGRA framework, with the parameter space defined by Eq. (6), obey this condition. On the other hand, Fig. 6 shows that in the general MSSM framework the relic density constraint creates a "hole" around $m_\chi \approx 100$GeV in the theoretically allowed region (coming from the annihilation channels into W bosons). Hence, there is a small window (namely $M_\chi < 70$ GeV) in which a low mass neutralino can have a very low value of σ. Furthermore, in order to change the sign of N_{14}, $|M_1/\mu|$ cannot be very small. Consequently, there is an upper bound on $|\mu|$ - the results of our code show roughly $|\mu| < 300$ GeV in models with $M_\chi < 70$ GeV and with low values of σ. Note that $\mu < 0$ is necessary for N_{14} to change sign. However, in the mSUGRA framework, the $b \to s + \gamma$ constraint does not allow $0 > \mu > -300$ GeV, hence preventing the cancellation from taking place. Examples of such models allowed in the MSSM, but not allowed in a mSUGRA framework can be seen in the final three lines of the Table 1.

Second, we consider models in the general MSSM framework with low σ values and $M_\chi > 150$ GeV. These models tend to have large $|\mu|$ and/or large scalar masses (for example, the first two rows in the Table 1). In this way, they violate the naturalness assumption of the mSUGRA framework (the constraint

$m_0 < 1$ TeV and $m_{1/2} < 350$ GeV in mSUGRA framework implied that the top squark mass is below ≈ 3 TeV). This remains true even when we relax to $m_{1/2} < 1$ TeV, but the safety margins are smaller. In other words, relaxing naturalness further in the mSUGRA framework would probably lead to models with the cancellation and, therefore, with lower values of σ.

5 Conclusion

We summarize our results as follows. The contributions to the cross section for spin independent elastic scattering of neutralinos off nucleons come from the exchange of the Higgs bosons and squarks. The contribution of the squark exchange is usually much smaller, being of importance only when Higgs boson exchange contribution is very small.

We investigate different conditions which could lead to small Higgs boson exchange contribution. We find that in mSUGRA framework, with the free parameter ranges defined in Eq. (6), these conditions are not satisfied primarily due to the relationship between parameters M_1 and μ (coming from the unification and radiative electroweak symmetry breaking assumptions), the naturalness assumption (which keeps different parameters from becoming very large) and the $b \to s + \gamma$ constraint (which removes most $\mu < 0$ models). We find that the light Higgs boson exchange dominates over the other channels and it leads to $\sigma > 10^{-46}\mathrm{cm}^2$. Equivalently, this yields an event rate > 0.1 ton^{-1}day^{-1} in ^{73}Ge target, which could be within reach of the future direct detection experiments. However, if we relax the naturalness constraint by a factor of three (in particular, the constraint on $m_{1/2}$), parameters can be tuned to satisfy one of the conditions (no light Higgs exchange) and this drives the lower bound on σ down by almost two orders of magnitude.

In the more general MSSM models (as defined in and above Eq. (31)), the situation is significantly different. The scattering of the Higgs bosons off of down-type quarks in the nucleus usually dominates, but, as discussed above, in some cases it can be relatively smaller and of the 'wrong' sign, so that it destructively interferes with the other parts of the amplitude, driving the total cross section very small. Similarly, dropping the naturalness constraints allows μ to be very large. This, in turn, implies that the neutralino is a very pure bino, hence removing the Higgs boson scattering channels and pushing the overall σ down. The direct detection of these models would be significantly harder.

6 Acknowledgements

VM thanks Bernard Sadoulet and Richard Gaitskell for the tremendous support and help during this work. PG thanks Bernard Sadoulet for hospitality at the CfPA. The work of HM and AP was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grant PHY-95-14797. AP is also supported by a National Science Foundation Graduate Fellowship. The work of VM was supported by the Center for Particle Astrophysics, a NSF Science and Technology Center operated by the University of California, Berkeley, under Cooperative Agreement No. AST-91-20005 and by the National Science Foundation under Grant No. AST-9978911.

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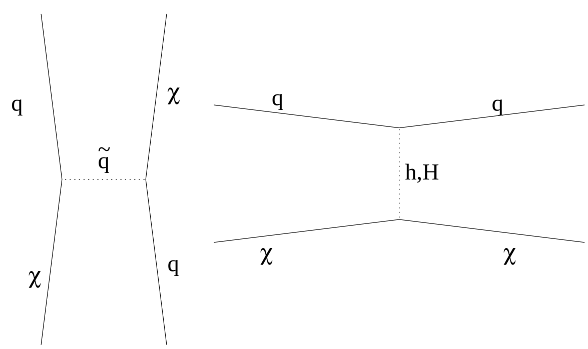

Figure 1: The leading diagrams for direct detection. Note that there is also a u-channel diagram for squark exchange. There are also diagrams where the neutralino scatters off of gluons in the nucleon through heavy squark loops.

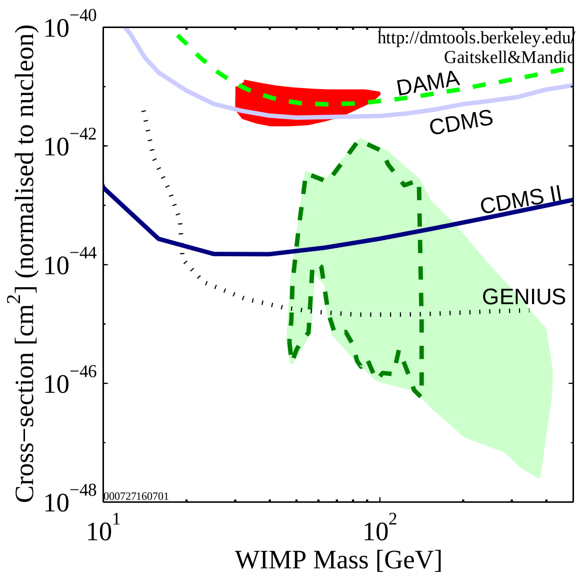

Figure 2: The cross section for spin-independent χ-proton scattering is shown. A relatively conservative relic density cut is applied, $0.025 < \Omega h^2 < 1$. Current accelerator bounds, including $b \to s + \gamma$ are imposed through the DarkSUSY code. The darker region is the DAMA allowed region at 3σ CL. The dashed curve is the DAMA 90% CL exclusion limit from 1996 (obtained using pulse-shape analysis). The lighter solid curve is the current CDMS 90% CL exclusion limit, the darker solid curve is the projected exclusion limit for CDMS II experiment and the dotted curve is the projected exclusion limit for the GENIUS experiment. The lighter shaded region is the result of this paper. It represents models allowed within the mSUGRA framework for the constraint $m_{1/2} < 1$ TeV. The closed dashed curve inside this region bounds the region of models allowed in the mSUGRA framework for the constraint $m_{1/2} < 350$ GeV.

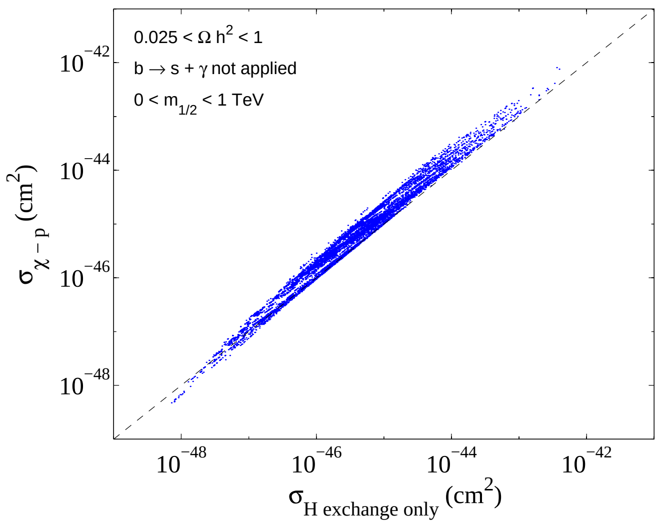

Figure 3: The cross section for spin-independent χ-proton scattering: the complete (DarkSUSY) calculation is shown on the y axis and the contribution to the cross section from the Higgs bosons exchange alone is shown on the x axis. A relatively conservative relic density cut is applied, $0.025 < \Omega h^2 < 1$, along with the constraint $0 < m_{1/2} < 1$ TeV (instead of $0 < m_{1/2} < 350$ GeV).

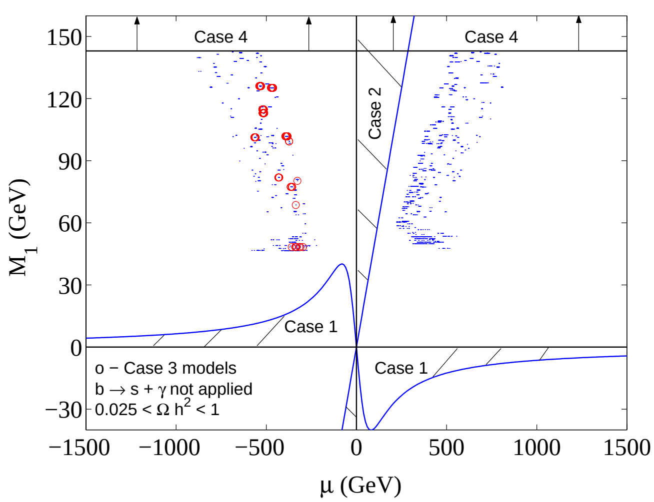

Figure 4: This figure illustrates the regions of $M_1 - \mu$ plane in which the 4 cases discussed in the text are satisfied. $b \to s + \gamma$ constraint was not applied, but relatively conservative relic density cut ($0.025 < \Omega h^2 < 1$) was applied, along with the constraint $0 < m_{1/2} < 350$ GeV.

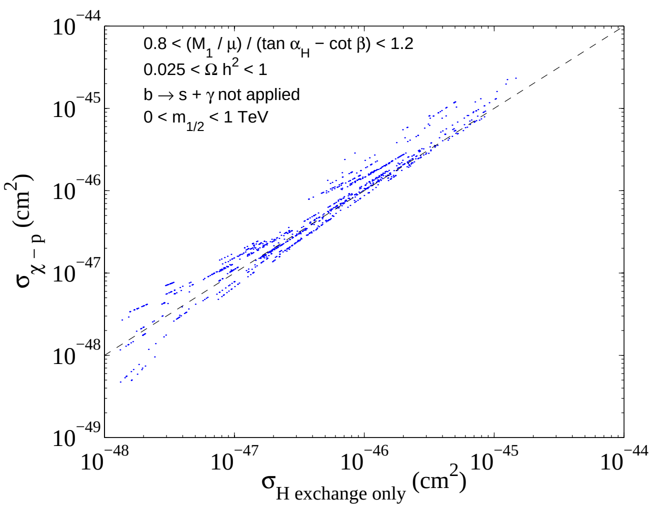

Figure 5: The proton-neutralino scattering cross section for models where the condition in Case 3 is approximately satisfied. The complete (DarkSUSY) calculation is shown on the y-axis and the contribution to the cross section due only to the exchange of the heavy Higgs boson, H, is shown on the x-axis. A relatively conservative relic density cut is applied, $0.025 < \Omega h^2 < 1$, along with the constraint $0 < m_{1/2} < 1$ TeV. The $b \to s + \gamma$ constraint (along with $0 < m_{1/2} < 350$ GeV) eliminates all models shown on this plot.

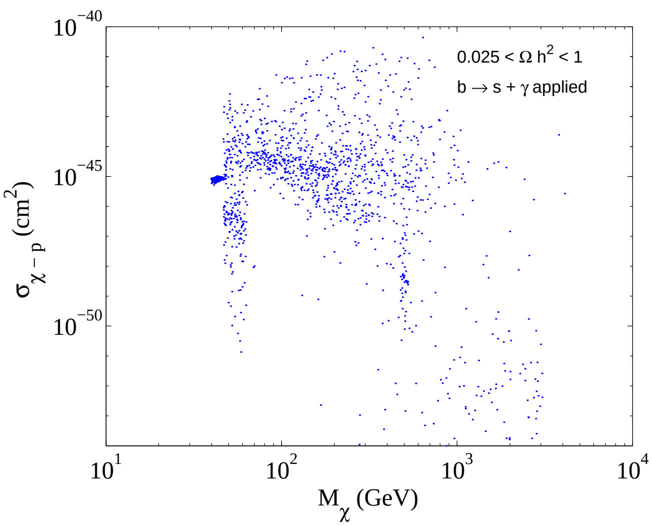

Figure 6: The cross section for spin-independent χ-proton scattering in the general MSSM framework is shown. A relatively conservative relic density cut is applied, $0.025 < \Omega h^2 < 1$. Note also that the constraint on the gaugino fraction, $z_g > 10$, is applied. Current accelerator bounds, including $b \rightarrow s+\gamma$ are imposed through the DarkSUSY code.

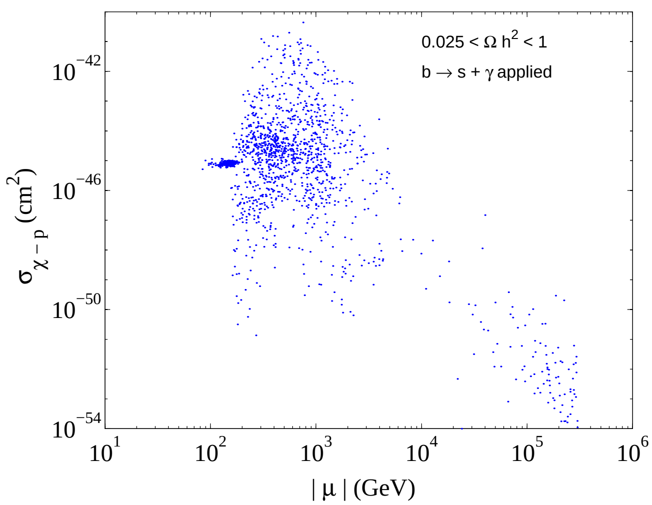

Figure 7: The cross section for spin-independent χ-proton scattering in the general MSSM framework is shown. A relatively conservative relic density cut is applied, $0.025 < \Omega h^2 < 1$. Note also that the constraint on the gaugino fraction, $z_g > 10$, is applied. Current accelerator bounds, including $b \to s + \gamma$ are imposed through the DarkSUSY code.